An Investigation of Beta and Downside Beta Based CAPM-Case Study of Karachi Stock Exchange

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Abstract

Sharpe’s (1964) Capital Asset Pricing Model (CAPM) assumes that the relationship between risk and return is positive, linear and significant. However, it is not free from controversies and one of them advocates replacing CAPM’s beta by downside beta based on investors’ preference of downside risk. Roy (1952) debates that investor care for downside risk and Hogan and Warren (1974) replace variance with semivariance in CAPM as the first official version of downside risk based CAPM. Bawa (1975), Fishburn (1977) and Bawa and Lindenberg (1977) develop and extend proxy for downside risk/beta as Lower Partial Moment. This study empirically tests beta and downside beta based CAPM (DCAPM). Conceptual and empirical problems related in testing alternative models are discussed with adoption of Fama-MacBeth (1973) procedure by making it robust. This study inspects intercept, risk-return relationship, nonlinearities and effect of residuals for both CAPM and DCAPM. Intercept results are almost similar and they follow introduction of zero-beta models as outlined by Black et al. (1972). Both models show rejection of nonlinearities and effect of residuals. However, DCAPM comes out to be strong contender compared to CAPM for risk-return relationship. These results are consistent with Estrada (2002), Ang et al. (2004) and Post and Vliet (2004).

Keywords:  CAPM, variance, downside risk, lower partial moments.
1. Introduction

A suitable asset pricing model for a stock market of a country has been an area of great interest for researchers, academicians, corporate managers and policymakers alike. Primarily, asset pricing models assess risk-return relationship in order to ensure value for the stakeholders in financial markets. Though there are more than one asset pricing models but Capital Asset Pricing Model developed by Sharpe (1964) with its foundation in Markowitz’s (1952) Modern Portfolio Theory is simple, cost-effective and most widely used asset pricing model in the world. Yet, it is most discussed topic in the field of financial economics and for the last half a century, major portion of the literature in the field of asset pricing is associated with the issues pertaining to CAPM. Fundamentally, it is assumed that the relationship between risk and return is positive, linear and significant. It is also assumed that investors are indifferent to upside and downside risks with stock returns being normally distributed (Sharpe (1964)). However, CAPM is not free from controversies and one of them advocates replacing CAPM’s beta by downside beta based on investors’ preference of downside risk.

Roy (1952) debates that investor care for downside risk, or simply, safety from disaster as foremost goal. Roy’s Safety First-rule is not used in asset pricing until Hogan and Warren (1974) replace variance with semivariance in CAPM. They give the first official version of downside risk based CAPM (DCAPM). Second breakthrough is made by Bawa (1975) who develops proxy for stochastic dominance as Lower Partial Moment (LPM) and later, Fishburn (1977) extends it into unlimited scope of LPM. Bawa-Fishburn LPM encompasses all classes of investors; risk-averse, risk-seeking and risk-neutral. Additionally, it is not tied to condition of normality and is flexible to include skewness and kurtosis as well. Moreover, Bawa and Lindenberg (1977) extend symmetric partial moments to asymmetric generalized co-LPM or GCLPM for n-degree LPM structures accommodating asymmetric distributions.

This study empirically tests both CAPM and DCAPM for the relationship between risk and return as positive, linear and significant. Section one is introduction and in section two, this study addresses issues concerning CAPM and DCAPM and then followed by their respective evidences. In section three, it discusses data and Fama-MacBeth (1973) procedure with robust analysis to study risk-return relationship as underlined by CAPM and DCAPM. Additionally, issues like nonstationary betas, unobservability of true market portfolio, measurement error, portfolio sorting procedures, heteroscedasticity and autocorrelation, investment horizon and equal-weighted portfolios are also covered. In section four, results are discussed and last section is conclusion.

2. Literature Review

2.1. CAPM-An Introduction

CAPM is independently developed by Sharpe (1964), Lintner (1965), Mossin (1966) and Treynor (1962). It is based on ideal assumptions and has roots in Markowitz’s Modern Portfolio Theory and
Tobin’s 2-Fund Separation Theorem. Although, CAPM does not depict prices but it is still regarded as an asset pricing model as one can determine and forecast prices of the securities in a financial market. It predicts positive, linear and significance relationship between risk and return and implies that stocks strongly covary with market and exhibit high returns in contemporaneous periods (Sharpe (1964)). It is based on ideal assumptions and some of them are stated henceforth.

CAPM assumes that investors have rational expectations which form the very basis of efficient market hypothesis (EMH). It also assumes Expected Utility Theory based on von Neumann–Morgenstern utility function (1953). Moreover, it is assumed that investors display three major characteristics; risk-averseness, nonsatiation and decreasing absolute risk averseness (DARA). Quadratic concave utility curve depicts investors’ behavior for the given three major characteristics (Elton et al. (2003)). However, these foundations of CAPM have been criticized and numerous solutions have been suggested. Some have completely rejected it while some argue presence of anomalies resulting in extension of single-factor CAPM into multi-factor models. And most importantly, theoretical shortcomings have been addressed and contributions towards the modified versions of CAPM have been made.

A meticulous analysis yields that it is important for the investors to define, identify and allocate value to risk as it. Theoretical framework of CAPM assumes that investors place equal weights to both upside and downside risk however this assumption is contentious. It has been observed that investors are willing to pay premium for stocks giving upside potential in bull markets and downside protection in bear markets with care for safety from disaster as foremost goal, or simply saying, they prefer to diversify downside risk. Moreover, under large departure from the condition of normality, distribution becomes severely asymmetric, thus, making difference between behavior for bear and bull markets more pronounced (Chunachinda et al. (1997)). It can be safely deduced from the above discussion that investors prefer downside risk and they are not limited to the class of risk-averse investors.

2.2. Downside Risk—An Introduction

Markowitz (1952) initially proposes expected return and semivariance, proxy of downside risk, as key parameters for portfolio investment decision. However, he discards semivariance due to resource

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6 Absence of transaction costs and personal income tax, assets can be divided infinitely, an individual cannot affect stock prices solely, investors take decisions on expected values and variance only, unlimited short sales allowed, all assets are marketable, unlimited lending and borrowing is at risk-free rate, homogeneity of expectations (Elton et al. (2003)).
7 Markowitz (1952) proposes that investors, in mean-variance (MV) framework, try to maximize return (E) for a given risk (V) or strive to minimize risk for a given return in his Modern Portfolio theory.
8 2-Fund Separation is presented by Tobin (1958) and purposes significant course of action to identify suitable portfolios in efficient set by introducing concept of risk-free asset with portfolio of risky assets. It states that an investor will strive for a utility-maximizing portfolio by combining risk-free and risky assets.
9 Rather it uses percentages.
10 *Ex ante* expected returns are unobservable, so instead *ex post* is used. To accomplish this, investor has rational expectations is assumed i.e. rational expectations purposes that outcomes are equal to expectations. Muth (1961) proposes rational expectation and Roberts (1959) endorses use of rational expectations in asset pricing. Fama (1965) assumes that investors are rational and all past information is incorporated in current prices of stocks leading to market efficiency.
11 EMH asserts that financial markets are “informational efficient”. An individual cannot consistently achieve stock returns in excess of average market returns. There are three major versions of EMH namely; weak, semi-strong, and strong form of efficiency (Haugen (2001)).
12 Expected Utility Theory primarily assumes that investors are rational and they choose and prefer an opportunity which maximizes expected value represented as a utility function.
13 For details see Abbas et al. (2011A) and Abbas et al. (2011B).
14 as they become are risk averse for losses and risk seekers for gains. For details see Ang et al. (2002).
constraints at that time. Roy (1952) debates that investor care for downside risk, or simply, safety from disaster as foremost goal. Roy’s (1952) Safety First-rule is not used in asset pricing until Hogan and Warren (1974) who replace variance with semivariance as the first official version of downside risk based CAPM. Second breakthrough is made by Bawa (1975) who develops proxy for downside risk as Lower Partial Moment (LPM) based on stochastic dominance. Later, Fishburn (1977) extends Bawa’s (1975) LPM into unlimited scope of LPMs. Bawa-Fishburn LPM encompasses all classes of investors; risk-averse, risk-seeking and risk-neutral. Additionally, it is not tied to condition of normality and is flexible to include skewness and kurtosis. Moreover, Bawa and Lindenberg (1977) extend symmetric partial moments to asymmetric generalized co-LPM or GCLPM for n-degree LPM structures accommodating asymmetric distributions.

Investors exhibit three types of behaviour namely; nonsatiation, risk-averseness and DARA (Post and Vliet (2004)). CAPM is restrictive to the class of risk-averse investors. Thaler and Johnson (1990), later supported by Levy and Levy (2002, 2004), report that investors are risk-seeking over gains and risk-averse over losses. Shefrin and Statman (2000) explore behavioral science and report that investors hold short and long position which depicts risk-averse and risk-seeking behavior. Hartley and Farrell (2002) advocate global convex utility function which indicates risk-seeking behavior. Secondly, CAPM’s quadratic utility tries to capture all the three types of behavior but fails in DARA’s domain (Lengwiler (2004)). It cannot exhibit DARA which is essential for risk aversion as it turns down upon reaching bliss point (Chavas (2004)).

Moreover, CAPM assumes condition of normality to qualify variance as a measure of investors’ perception of risk (Sharpe (1964)). Contrarily, stock returns are found not to exhibit normality. Fama (1965), Kon (1984), Affleck-Graves and McDonald (1989), Richardson and Smith (1993), Aparicio and Estrada (1997), Susmel (2001), Hwang and Pedersen (2002) and Dufour et al. (2003) provide evidence that returns are not normally distributed for developed market. Bekker and Harvey (1995, 1997), Eftekhari and Satchell (1996) and Bekkert al. (1998) conclude that emerging market equities demonstrate non-normality. Under large departure from normality, probability distribution is severely asymmetric in which CAPM does not hold (Chunachinda et al. (1997), Athayde and Flôres (2004) and Jondeau and Rockinger (2006)).

Most notably, CAPM assumes that investors are indifferent towards upside and downside risk while evidence is present where investors portray different behavior during bear and bull markets. They are willing to pay premium for stocks giving downside risk protection in bearish trends. Roy (1952) advocates that investors care for downside risk only. Markowitz (1959) introduces semivariance in portfolio theory as a possible proxy of risk. Kahneman and Tversky (1979), Tversky and Kahneman (1992), Gul (1991), and Estrada (2000, 2002 and 2007) are supportive of different behaviors of investors for bear and bull markets. Post and Levy (2005) stress on preference of downside risk over variance as a measure of investors perception of risk.

Bawa-Fishburn-Lindenberg based-LPM addresses these theoretical weaknesses of CAPM. It is not confined to DARA as n-order LPMs are used to envelop wide range of investor’s risk measures defined as:

\[ \beta_{D} = \frac{\text{cov}(R_i, R_M | R_M < \tau)}{\text{var}(R_M | R_M < \tau)} \]
\[ LPM_{in}(\tau, R_i) = \int_{-\infty}^{\tau} (\tau - R_i) \]

Where \( \tau \) is the target return specified by investor, \( R_i \) is return of asset \( i \), \( dF(R_i) \) is probability density function of asset \( i \) and \( n \) is order of investor’s risk preference for \( \text{ex ante} \). Setting \( LPM=1 \) or \( n=1 \) indicates that risk-neutral investor. For \( n>1 \), investor is risk-averse and \( n<1 \) depicts risk-taker. For \( n=2 \), investor is semivariance conscious risk-averse. 3 and 4 depict skewness and kurtosis, respectively.

Bawa-Fishburn-Lindenberg based-LPM has four distinct advantages over CAPM framework. Firstly, it encompasses all the classes of investors; risk-averse, risk-seeking and risk-neutral; and incorporates skewness and kurtosis in equation (1) for \( n \) values of 3 and 4 respectively. Secondly, it can use fractional degrees like 2.33 or 3.89 for different \( n \) making Bawa-Fishburn-Lindenberg based-LPM more receptive to sensitivity analysis. Thirdly, it introduces asymmetric covariance as generalized or asymmetric co-LPM or GCLPM for \( n \)-degree LPM structures\(^{21}\) in which covariance between securities \( i \) and \( j \) is not necessarily equal to covariance between \( j \) and \( i \) (Bawa (1975), Fishburn (1977), Bawa and Lindenberg (1977), Nawrocki (1991, 1999), Harlow (1991), Estrada (2002) and Abbas et al. (2011A)).

Lastly, Bawa-Fishburn-Lindenberg based-LPM introduces downside risk as a measure of investor’s perception of risk. \( \tau \) is the target return which is specified by an investor. \( \tau \) is flexible as it can take any value depending on the preference of the investor. Bawa and Lindenberg (1977) suggest that \( \tau \) is equal to risk-free interest rate. However, later versions assume \( \tau \) for any value according to investor’s choice (Nantell and Price (1979), Harlow and Rao (1989), Harlow (1991) and Estrada (2002)). These distinct advantages make Bawa-Fishburn-Lindenberg based-LPM an excellent choice as a single asset pricing model. Moreover, it does not contravene CAPM’s assumption especially Tobin’s (1958) 2-Fund Separation Theorem (Hogan and Warren (1974), Bawa and Lindenberg (1977), Harlow and Rao (1989) and Estrada (2002)), However, evidence for both variance based CAPM and downside risk based LPM single asset model\(^{22}\) has to be investigated.

2.3. CAPM and its Evidence


In Asian markets and emerging markets, results for CAPM are mixed. Aggarwal et al. (1988), Chan et al. (1991), Cheung and Wong (1992), Daniel et al. (1997), Huang (1997), Chui and Wei (1998) and Hodoshima et al. (2000) report for different Asian and emerging markets that the risk-return relationship as underlined by CAPM does not hold. Contrarily, Ariff and Johnson (1990), Gillan (1990), Kim et al. (1992), Ma and Shaw (1990), Wong and Tan (1991), Xu (2001) and Chen (2003) report evidence supporting CAPM for different Asian and emerging markets. These results are not conclusive pertaining to suggest the trend of findings of CAPM for different Asian and emerging markets.

\(^{21}\) For symmetric returns, semivariance criteria do not differ from variance, however, if returns show asymmetric pattern then covariance between securities \( i \) and \( j \) is not equal to covariance between \( j \) and \( i \) unlike CAPM’s covariance which assumes equality in either case (Nawrocki (1991, 1999)).

\(^{22}\) From now DCAPM.
2.4. Downside Risk and its Evidence

Evidence for downside risk is mostly concentrated in the developed markets. Emerging markets have been only given more attention to downside risk in the last decade or so. Quirk and Saposnik (1962), Mao (1970), Klemkosky (1973), Hogan and Warren (1974), Bawa (1975), Harlow and Rao (1989), Harlow (1991), Grootveld and Hallerbach (1999), Harvey (2000), Balzer (2001), Ang et al. (2002, 2006), Post and Vliet (2004) and Bali et al. (2009) report that downside risk holds in different developed markets. Moreover, it is also reported that downside risk based CAPM (DCAPM) performs better compared to variance based CAPM.

Estrada (2002), Estrada and Serra (2005), Galagedera and Brooks (2007) and Galagedera and Jaapar (2009) report their findings in favour of downside risk for different emerging markets. However, emerging markets provide a more challenging ground as these markets are not developed and have characteristics of high volatility and turnover. Another characteristic of emerging markets is that they are peculiar and they do not show uniformity of characteristics among them23. However, due to feature of high volatility and turnover, Bawa-Fishburn-Lindenberg based-LPM, proxy of stochastic dominance, is more adoptive to this feature as stochastic dominance does not rely on a certain type of distribution function.

3. Data and Methodology

3.1. Data

First major study in KSE-Pakistan for CAPM is by Ahmad and Zaman in 1999. They use GARCH-M model to test the risk-return relationship and conclude that stock return follow cyclical trend. Ahmad and Qasim (2004) claim that positive shocks are more pronounced compared to negative ones for same market. Iqbal and Brooks (2007) report presence of non-linear risk-return relationship. Javid and Ahmad (2008) test CAPM in conditional settings and advocate use of conditional-CAPM compared to conventional CAPM. Recently, Iqbal et.al. (2010) compare single-factor CAPM with Fama-French multi-factor CAPM and conclude that the latter is more convincing for Pakistan market. However, all of them do not address issue of investors’ preference of downside risk and are restrictive to variance as a measure of risk.

This study chooses Karachi Stock Exchange (KSE)-Pakistan as a sample. KSE24 is an important25 emerging market showing typical characteristics of high turnover26 and high price volatility27. 84 listed-companies listed in KSE are taken as a sample with a estimation window from 2000 to 2010 as the history of KSE-100 index is short. Only those companies are selected which are listed throughout this period following Javid and Ahmad (2008). To validate results, our empirical analysis encompasses monthly data as underlined by previous studies like Fama and MacBeth (1973) and Fama and French (1992).

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23 www.MSCI.com
24 Total of 651 companies are listed with KSE having a market capitalization of $35 billion on July 30,2011.
25 Due to strong institutions like KSE, Pakistan’s economy has been included in four emerging markets list by Dow Jones. This list represents those economies that are expected to contribute to global growth. This shows the confidence that institutions like KSE-Pakistan have future growth potential.
26 KSE experiences a high turnover and high price volatility during the last decade. Despite the small size of the market, it experiences high turnover of 323% as compared to NYSE having turnover 65% in 2006. KSE-100 index rises by 650% from 2001 to 2005 while Bombay (Mumbai) Stock Exchange (BSE30) experiences a rise of 137% for the same period.
27 KSE is declared as Best Performing Stock Market of the World by Business Week in 2002. Bullish trend appears to have developed from 2002 to 2005 and is busted in March 2005. However, KSE recovers and crosses 15,000 barrier first time in its history in April 2008. However, in May 2008, SBP increases interest rates unexpectedly to address high inflation, KSE crashes and consequently, in August, 2008 the floor is halted. Afterwards, trading is allowed to resume on December 15, 2008 and amazingly, KSE recovers 20-25% of its decline in index till March 12, 2009.
3.2. Methodology-Fama-MacBeth Procedure with Robustness

Fama MacBeth,\(^\text{28}\) (1973) procedure is most widely used and historically important method to test CAPM. It widespread acceptance is primarily motivated to fact that FMac (1973) allow betas to vary with time\(^\text{29}\) (Campbell et al.(1997) and Elton et al.(2003)). FMac (1973) procedure also caters for measurement error by building portfolios in a peculiar way\(^\text{30}\). Moreover, they use beta of previous time period as an instrumental variable to cater for selection bias \(^\text{31}\). These reasons make FMac (1973) two-step approach an obvious choice to test CAPM and DCAPM. FMac (1973) use two-step approach by forming pre-ranking beta portfolios and testing for post-ranking beta portfolios. This study adopts FMac (1973) procedure and makes necessary changes to better serve the results of this study. Firstly, first pass\(^\text{32}\) of FMac (1973) procedure is performed which is based on the Black, Jensen and Scholes (1972) methodology as follows:

\[
R_{it} - R_{Ft} = \alpha_i + \beta_i (R_{Mt} - R_{Ft}) + \epsilon_{it}
\]

where \(R_{it}\) is return of stock \(i\) at time \(t\), \(R_{Ft}\) is return of treasury bills, \(\alpha_i\) is y-intercept, \(\beta_i\) is beta of stock \(i\) and \(\epsilon_{it}\) is the error term. It is supposed that mean of residuals of asset \(i\) at time \(t\) is zero; \(E(\epsilon_{it}) = 0\) and covariance between risk premium and residuals is also equal zero; \(\text{cov}(R_{Mt}-R_{Ft}, \epsilon_{it}) = 0\).

Firstly, this study estimates betas of the respective stocks and the first eighty stocks are selected. 40 stock portfolios are created using beta estimates and these portfolios are resorted based on downside beta to yield 20 stock portfolios based Fama MacBeth (1973)\(^\text{33}\). In all, 82 portfolios are built and ranked from highest to lowest downside beta portfolios. This procedure is repeated again by firstly sorting stocks on downside beta into portfolios and then resorts these portfolios on beta basis to yield 82 portfolios ranked from highest to lowest beta portfolios. These two sets of portfolios of 82 each are used in first pass as defined in equations (1) to yield beta and downside beta of portfolios for the former and the latter set respectively.

Moreover, stocks are sorted on relative downside beta to form 82 portfolios to perform FMac (1973) regressions to make sure that downside beta is not reflecting regular beta. Moreover, incremental effect of downside beta can be assessed. Relative DB is defined as the difference between \(\tilde{\beta}\) regressions to make sure that downside beta is not reflecting regular beta. This approach has two major advantages namely; it specifies sorting criteria and\(^\text{34}\) decreases and disappear frequently when event studies are based on value-weighted portfolios. Secondly, Harvey and Siddique (2000) assert that coskewness is strongest for equal-weighted portfolios. Thirdly, Roll (1981), Ohlson and Rosenberg (1982), Grinblatt and Titman (1989) and Korajczyk and Sadka (2004) conclude that equal-weighted portfolios

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\(^{28}\)From now FMac.

\(^{29}\)Testing CAPM is difficult as betas are non-stationary over time. Fama and MacBeth (1973) propose flexible solution to allow for time-varying risk measures with time-varying risk premiums. This approach is most appealing as it depicts real data more correctly when it is used in rolling regression.

\(^{30}\)Friend and Blume (1970), Black et al.(1972) and Fama and MacBeth (1973) improve precision of estimated betas by working on portfolios rather than individual stocks. Compared to beta estimates for individual stocks, estimates for portfolio beta lead to reduction in measurement error.

\(^{31}\)Grouping procedure reduces statistical power of estimated betas by shrinking them. To cater for this, an instrumental variable is introduced which is highly correlated with true beta and can be independently observed thus resolving problem of selection bias ((Black et al.(1972), Fama and MacBeth (1973), Huang and Litzenberger (1988) and Elton et al.(2003)).

\(^{32}\)In first pass, time series estimation is performed in which portfolio returns are regressed against market returns to obtain risk measures like beta and downside beta.

\(^{33}\)Chen and Martin (1980) and Tole (1981) report that the larger the size of portfolio, the stable the beta will be of that portfolio. Thus portfolio size is directly proportional to stationary of respective portfolios betas. Elton and Gruber (1977) advocate 10-15 stocks, Evans and Archer (1968) propose that around 10 stock portfolio, Fama MacBeth (1973) propose 20 stock portfolio and Statman (1987) asserts 30-40 stock portfolio to be an appropriate portfolio size. This study uses 20-stock portfolios as trade-off point between portfolio size and diversification following Fama MacBeth (1973).

\(^{34}\)For details see Post and Vliet (2004) and Ang, Chen and Xing (2004).

\(^{35}\)Equal-weighted are preferred over value-weighted portfolios; firstly, Fama (1998) concludes that anomalies seem to decrease and disappear frequently when event studies are based on value-weighted portfolios. Secondly, Harvey and Siddique (2000) assert that coskewness is strongest for equal-weighted portfolios.
estimator adjusted GMM for problem of heteroscedasticity and autocorrelation in FMac (1973) first pass for estimation window of 48 months. This makes estimated parameters more robust as they are corrected for heteroscedasticity and autocorrelation. For cross-sectional second pass, this study use White estimator to make the t-stat robust. Choice of appropriate lags in GMM and Newey-West is important. GMM with three lags for each dependent and independent variable along with intercept is most appropriate that is used in this study. All results are estimated in Eviews 5 and 7.

Subsequently, FMac (1973) perform second pass for each month via cross-sectional analysis of portfolio beta and portfolio return. And eventually, they test for intercept and portfolio beta parameter, non-linearity and idiosyncratic underlying framework for their famous two-pass regression approach. To study relationship between expected return and beta, FMac (1973) use following equation for this purpose:

\[
R_p = \hat{\lambda}_0 + \hat{\lambda}_1 \beta_p + \hat{\lambda}_2 \beta_p^2 + \epsilon_{pt}
\]

(3)

Where \( R_p \) is portfolio returns, \( \hat{\lambda}_0 \) is intercept, \( \hat{\lambda}_1 \) depict risk-return relationship, \( \beta_p \) is portfolio beta and \( \epsilon_{pt} \) residuals for portfolio \( p \) at time \( t \).

Nonlinearity is determined by adding \( \beta_p^2 \) as additional term for three-dimensional axis with \( R_p \) on y-axis while \( \beta_p \) and \( \beta_p^2 \) on other two axis. It is added as follows:

\[
R_p = \hat{\lambda}_0 + \hat{\lambda}_1 \beta_p + \hat{\lambda}_2 \beta_p^2 + \hat{\lambda}_3 \beta_p^3 + \epsilon_{pt}
\]

(4)

For residual variance, another term \( RV_p \) is adding to check whether it affects stocks prices or not. The relationship is shown as:

\[
R_p = \hat{\lambda}_0 + \hat{\lambda}_1 \beta_p + \hat{\lambda}_2 \beta_p^2 + \hat{\lambda}_3 \beta_p^3 + RV_p + \epsilon_{pt}
\]

(5)

where \( RV_p \) is residual variance of portfolio \( p \).

In last, all parameters are added and their combine effect is measured and reported defined as:

\[
R_p = \hat{\lambda}_0 + \hat{\lambda}_1 \beta_p + \hat{\lambda}_2 \beta_p^2 + \hat{\lambda}_3 \beta_p^3 + RV_p + \epsilon_{pt}
\]

(6)

where is defined as \( RV_p = \sum_{j=1}^{M} \sigma^2(\epsilon_{j}) / M \) with an \( M \) is number of stocks in a portfolio \( j \).

FMac (1973) estimate their coefficient statistics for specific lamdas as:

have higher returns than counter-part. Lastly, absence of linear relationship has been attributed to smaller stocks (Ang et al. (2004)).
\[ \tilde{\lambda}_j = \frac{1}{T} \sum_{t=1}^{T} \hat{\lambda}_{jt} \]  \hspace{1cm} (7)

and

\[ \tilde{\sigma}^2(\tilde{\lambda}_j) = \frac{1}{T(T-1)} \sum_{t=1}^{T} (\hat{\lambda}_{jt} - \tilde{\lambda}_j)^2 \]  \hspace{1cm} (8)

and then they form t-stat as:

\[ t\left(\tilde{\lambda}_j\right) = \frac{\tilde{\lambda}_j}{\tilde{\sigma}(\tilde{\lambda}_j)} \]  \hspace{1cm} (9)

They calculate t-stat using (8) and consequently test hypothesis \( H_{01} \) as \( \tilde{\lambda}_0 = 0 \); intercept is not different from zero with restriction tested for two-sided test. \( H_{02} \) as \( \tilde{\lambda}_1 = 0 \); there are no nonlinearities in SML with restriction tested for two-sided test. \( H_{03} \) as \( \tilde{\lambda}_3 = 0 \); residual risk does not affect returns with restriction tested for two-sided test. \( H_{04} \) as \( \tilde{\lambda}_1 > 0 \); there is risk-return relationship with restriction tested for one-sided test.

\( H_{01} \) advocates that the intercept is not different from zero as FMac (1973) takes excess returns defined as difference of stock returns and risk-free rate. A failure to reject means preference by zero-beta model. Rejection of \( H_{02} \) means that risk-return relationship is linear as implied by the theory. \( H_{03} \) commits to notion that market operates as a fair game. And \( H_{04} \) is most important for positive risk-return relationship. This study adopts FMac (1973) procedure and estimates equations 5.2 through 5.5 for beta to study relationship between expected return and beta. Then it replaces beta by downside beta and again estimates equations 5.2 through 5.5 to study relationship between expected return and downside beta. In the end, this study introduces relative downside beta as measure of risk and estimates equations 5.2 through 5.5 to study relationship between expected return and relative downside beta.

4. Results for Fama and MacBeth Procedure
4.1. Fama and MacBeth Portfolios Sorted on Downside Beta/Beta

Examining Panel A of Table 1\(^{43}\), this study observes that intercept is rejected for CAPM at 10% significance level for both full sample and subsample of 2007-08 and 2009-10. However, \( \lambda_1 > 0 \) cannot be rejected. DCAPM also yield similar results but its intercept is rejected at 5% significance level. This study can safely say that both CAPM and DCAPM show positive risk-return relationship but intercept is significantly different from zero implying replacing of regular single asset pricing model to zero-beta/DB asset pricing model. Panel B reports results for intercept, risk-return relationship and nonlinearities in SML for both CAPM and DCAPM. Intercept is rejected in subsample of CAPM at 10% significance level while it is rejected at 1% significance level for total sample and at 5% for two subsamples. \( \lambda_1 \) is rejected in subsample of CAPM at 10% significance level but not for DCAPM. There are no nonlinearities in both the models. This study can safely say that DCAPM show positive risk-return relationship but CAPM misses at 10% significance level. Following results of Panel A, zero-beta/DB asset pricing model is advocated for intercept problem.

Table 1 reports results for intercept, risk-return relationship and effect of residuals on returns in Panel C. \( \lambda_0 \) is not rejected at any level of significance for CAPM but it is rejected at 1% significance level for full sample and 5% and 10% for subsamples for DCAPM. \( \lambda_1 \) is not rejected for any level for both models. \( \lambda_3 \) is rejected for DCAPM for subsample of 2007-08 at 10% significance level showing residuals affecting stock returns. This study can safely say that both CAPM and DCAPM show positive risk-return relationship however, DCAPM shows residual effect which is further investigated. Panel D shows results for all parameters; intercept risk-return relationship, nonlinearities and effect of residuals. Intercept follows similar trend as identified in previous panels. CAPM reveal nonlinear relationship for

\(^{43}\) All tables are given in appendix A.
subsample 2009-10 at 10% level. Residual effect is safely assumed to be negligible especially for DCAPM which shows some sign in Panel D. \( \lambda_1 > 0 \) holds for both CAPM and DCAPM but only to be replaced by their respective zero-beta versions. However, beta is also contributing to DB and its effect has to be nullified. So this study uses relative DB in Fama-MacBeth regressions to make the results robust and to observe incremental effects of DB.

### 4.2. Fama and MacBeth Portfolios Sorted on Relative Downside Beta

In Panel A of Table 2; this study observes that intercept is rejected for CAPM at 5% significance level for full sample and at 1% for subsample. However, \( \lambda_1 > 0 \) is rejected for CAPM at 10% level for total sample and 5% for subsample. DCAPM also yield similar result for rejection of intercept but \( \lambda_1 > 0 \) holds for it. This study can safely say that DCAPM show positive risk-return relationship but intercept is significantly different from zero. However, CAPM does not hold for the sample used in this study for significance level of 1% and 5%\(^{44} \). Table 2, Panel B reports results for intercept, risk-return relationship and nonlinearities in SML for both CAPM and DCAPM. Intercept is rejected in total as well as subsample of both CAPM and DCAPM. \( \lambda_1 \) is rejected in total sample of CAPM and subsample of DCAPM at 10%. There are no nonlinearities in both the models. This study can comment that both models show deviation at 10% significance level. However, this study can also comment that 5% level is not breached by both models in Panel B.

Panel C reports results for intercept, risk-return relationship and effect of residuals on returns. \( \lambda_0 \) is rejected at 5 and 10% level of significance for CAPM and DCAPM in different samples. \( \lambda_1 \) is rejected at 10% for CAPM in subsample and at same level for DCAPM in total sample. \( \lambda_3 \) is not rejected for either models paving the way for negligible effect of residuals on stock returns. This study can comment that both models show deviation at 10% significance level for their risk-return relationship. However, this study can also comment that 5% level is not breached by both models in Panel C following Panel B results. Panel D shows results of all parameters; intercept risk-return relationship, nonlinearities and effect of residuals. Intercept follows similar trend as identified in previous panels. \( \lambda_0 \) is rejected at 5 and 10% level of significance for CAPM and DCAPM for complete sample. For both CAPM and DCAPM, \( \lambda_1 \) is not rejected at any level of significance in total sample as well as in subsample. \( \lambda_2 \) and \( \lambda_3 \) are not rejected for either models paving the way for the absence of nonlinearities and negligible effect of residuals on stock returns.

Summing up, results for incremental effect of DB advocate DCAPM as strong contender for risk-return relationship to hold and CAPM being the weaker one for rejecting \( \lambda_1 > 0 \) at 5% level of significance. However, if this study chooses 10% level of significance, then DCAPM is not winner but relatively it is better. Secondly, the rejection of \( \lambda_0 = 0 \) is a common feature in all results which favours zero-versions of asset pricing models. This result is consistent with earlier studies of Black (1972), Stambaugh (1982), Gibbons (1982) and Shanken (1985).

### 5. Summary

Capital Asset Pricing Model (CAPM) is simple, cost-effective and most widely used asset pricing model in the world. Yet, it is most discussed topic in the field of financial economics\(^{45} \). It predicts positive and linear relationship between risk and return and assumes that stock returns are normally distributed. It is assumed that investors are indifferent to upside and downside risks (Sharpe (1964)). Roy (1952) debates that investor care for downside risk, or simply, safety from disaster as foremost goal defined as Safety First-rule. Bawa (1975) who develops proxy for Roy’s Safety-First rule based on stochastic dominance as Lower Partial Moment (LPM). Later, Fishburn (1977) extends it into...

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\(^{44}\) This result is consistent with findings of previous studies like Javid and Ahmad (2008) and Iqbal et.al. (2010).

\(^{45}\) Graham and Harvey (2001) report 73.5% of CFOs US and Brounen, Abe de Jong and Koedijk (2004) report 45% in Europe use CAPM.
unlimited scope of LPM and Bawa and Lindenberg (1977) extending it to asymmetric LPMs. Bawa-Fishburn-Lindenberg LPM encompasses all classes of investors; risk-averse, risk-seeking and risk-neutral. Additionally, it is not tied to condition of normality and is flexible to include skewness and kurtosis as well. When incorporated in CAPM framework, Bawa-Fishburn-Lindenberg LPM replaces regular beta by downside beta as downside risk based CAPM or DCAPM.

This study empirically tests beta based CAPM and downside beta based DCAPM. First this study discusses conceptual and empirical problems related to these models. As DCAPM has similar assumptions as of CAPM, so this study sticks to basic underlying methodology of testing CAPM i.e. Fama-MacBeth (1973) procedure. This study inspects intercept, risk-return relationship, nonlinearities and effect of residuals for both CAPM and DCAPM. Intercept results are almost similar and they advocate and follow rejection of null hypothesis. CAPM and DCAPM hold similar ground pertaining to nonlinearities and effect of residuals. Our results also reveal that risk-return relationship holds up, however, DCAPM comes out to be strong contender compared to CAPM for risk-return relationship. These results are consistent with Grootveld and Hallerbach (1999), Harvey (2000), Estrada (2002), Ang \textit{et al.}(2004) and Post and Vliet (2004). Intercept rejection leads us to zero-beta single asset pricing model as outlined by earlier studies of Black (1972), Stambaugh (1982), Gibbons (1982) and Shanken (1985).

This study can conclude and advocate the replacement of variance by downside risk as a suitable risk measure in a single asset pricing model. Downside risk based single asset pricing model will help in pricing securities more appropriately and anticipating portfolio risk for investors. Downside risk framework will also aid in determining the nature of investment. It will determine market behavior in a better way as compared to other single asset pricing models. DCAPM can be pivotal as key determinant for institutional demand for KSE stocks. Moreover, delineating cost of capital for capital budgeting is necessary to finance them. DCAPM can also contribute in this area leading to a suitable model of financing.

However, no study is complete and accordingly, this study recommends suggestions for future researchers. This study is primarily limited to monthly data and can be easily extended to daily and weekly observations. Use of conditional and multi-factor models in downside risk framework, not used in this study, will perhaps enhance the performance of DCAPM. Tracking Error in downside risk framework, if introduced, can make the model multi-dimensional and more receptive to market forces. Finally, behavioral approach can be incorporated to asset pricing as this field is steadily gaining attention among researchers and practitioners alike. This world still lays unexplored with vast frontiers to conquer.

References
An Investigation of Beta and Downside Beta Based CAPM-Case
Study of Karachi Stock Exchange


### Appendix A

#### Table 1: Results for Fama-MacBeth Regression based on Beta/DB Sorting

<table>
<thead>
<tr>
<th>Panel A</th>
<th>$R_{Pt} = \lambda_0 + \beta_P \lambda_1 + \epsilon_{Pt}$</th>
<th>$R_{Pt} = \lambda_0 + D\beta_P \lambda_1 + \epsilon_{Pt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_0$</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>2005-10</td>
<td>0.014*** (-1.921)</td>
<td>-0.001 (-0.560)</td>
</tr>
<tr>
<td>2005-06</td>
<td>0.006 (0.513)</td>
<td>0.001 (0.017)</td>
</tr>
<tr>
<td>2007-08</td>
<td>-0.023*** (-1.781)</td>
<td>-0.005 (-0.795)</td>
</tr>
<tr>
<td>2009-10</td>
<td>-0.026*** (-1.783)</td>
<td>0.001 (0.167)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B</th>
<th>$R_{Pt} = \lambda_0 + \beta_P \lambda_1 + \beta_P^2 \lambda_2 + \epsilon_{Pt}$</th>
<th>$R_{Pt} = \lambda_0 + D\beta_P \lambda_1 + D\beta_P^2 \lambda_2 + \epsilon_{Pt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_0$</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>2005-10</td>
<td>-0.012 (-1.409)</td>
<td>-0.003 (-0.635)</td>
</tr>
<tr>
<td>2005-06</td>
<td>0.005 (0.442)</td>
<td>-0.001 (-0.123)</td>
</tr>
<tr>
<td>2007-08</td>
<td>-0.011 (-0.644)</td>
<td>-0.018*** (-1.576)</td>
</tr>
<tr>
<td>2009-10</td>
<td>-0.031*** (-1.886)</td>
<td>0.009 (1.292)</td>
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</table>

<table>
<thead>
<tr>
<th>Panel C</th>
<th>$R_{Pt} = \lambda_0 + \beta_P \lambda_1 + \beta_P^2 \lambda_2 + \epsilon_{Pt}$</th>
<th>$R_{Pt} = \lambda_0 + D\beta_P \lambda_1 + D\beta_P^2 \lambda_2 + \epsilon_{Pt}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\lambda_0$</td>
<td>$\lambda_1$</td>
</tr>
<tr>
<td>2005-10</td>
<td>-0.014 (-1.456)</td>
<td>-0.003 (-0.651)</td>
</tr>
<tr>
<td>2005-06</td>
<td>0.004 (0.374)</td>
<td>-0.001 (-0.411)</td>
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<tr>
<td>2007-08</td>
<td>-0.024 (-1.267)</td>
<td>-0.007 (-0.598)</td>
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<tr>
<td>2009-10</td>
<td>-0.021 (-1.259)</td>
<td>-0.001 (-0.129)</td>
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</table>

<table>
<thead>
<tr>
<th>Panel D</th>
<th>$R_{Pt} = \lambda_0 + \beta_P \lambda_1 + \beta_P^2 \lambda_2 + \epsilon_{Pt}$</th>
<th>$R_{Pt} = \lambda_0 + D\beta_P \lambda_1 + D\beta_P^2 \lambda_2 + \epsilon_{Pt}$</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>$\lambda_1$</td>
</tr>
<tr>
<td>2005-10</td>
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<td>0.001 (-0.077)</td>
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<tr>
<td>2005-06</td>
<td>0.001 (0.117)</td>
<td>0.000 (0.153)</td>
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<tr>
<td>2007-08</td>
<td>-0.011 (-0.517)</td>
<td>-0.015 (-1.199)</td>
</tr>
<tr>
<td>2009-10</td>
<td>-0.038*** (-2.201)</td>
<td>0.013 (1.618)</td>
</tr>
</tbody>
</table>

Results for cross-sectional analysis are given for intercept, risk-return relationship, nonlinearities and effect of residuals for column 2 and 3 and 6 and 7 respectively for beta/DB sorted portfolios. Results for full sample are given in bold from 2005 to 2010. Results for three subsamples are also given for breakup purpose. *t-stat are given in brackets. * shows significance at 1 percent, ** shows significance at 5 percent and *** shows significance at 10 percent.
Table 2: Results for Fama-MacBeth Regression based on DB/Beta Sorting

<table>
<thead>
<tr>
<th>Panel</th>
<th>( R_{Pt} = \lambda_0 + D\beta_{Pt} \lambda_1 + \varepsilon_{Pt} )</th>
<th>( R_{Pt} = \lambda_0 + D\beta\beta_{Pt} \lambda_1 + \varepsilon_{Pt} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( \lambda_0 )</td>
<td>( \lambda_1 )</td>
</tr>
<tr>
<td>2005-10</td>
<td>-0.024**</td>
<td>-0.004***</td>
</tr>
<tr>
<td></td>
<td>(-2.695)</td>
<td>(-1.495)</td>
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<tr>
<td>2005-06</td>
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<td>-0.0002</td>
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<tr>
<td></td>
<td>(-3.076)</td>
<td>(-0.032)</td>
</tr>
<tr>
<td>2007-08</td>
<td>-0.019</td>
<td>-0.0055</td>
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<tr>
<td></td>
<td>(-1.199)</td>
<td>(-0.372)</td>
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<tr>
<td>2009-10</td>
<td>-0.020</td>
<td>-0.006</td>
</tr>
<tr>
<td></td>
<td>(-1.398)</td>
<td>(-0.953)</td>
</tr>
</tbody>
</table>

Results for cross-sectional analysis are given for intercept, risk-return relationship, nonlinearities and effect of residuals for column 2 and 3 and 6 and 7 respectively for DB/Beta sorted portfolios. Results for full sample are given in bold from 2005 to 2010. Results for three subsamples are also given for breakup purpose. t-stat are given in brackets. *shows significance at 1 percent, ** shows significance at 5 percent and *** shows significance at 10 percent.