CAPM-Exclusive Problems Exclusively Dealt

Dr. Qaiser Abbas (Corresponding Author)
Professor
Department of Management Sciences
COMSATS Institute of Information Technology,
Chak Shahzad, Park Road, Islamabad.

Usman Ayub
Assistant Professor and PhD. (Scholar)
COMSATS Institute of Information Technology,
Chak Shahzad, Park Road, Islamabad.

Syed Kashif Saeed
PhD. (Scholar)
COMSATS Institute of Information Technology,
Chak Shahzad, Park Road, Islamabad.

Abstract
The Capital Asset Pricing Model allows to price risky financial assets, in seductive simple way, but under various theoretical assumptions. Since its inception, CAPM has been questioned due to some of its unrealistic theoretical assumption or due to its empirical failures. Academicians have been concentrating to minimize its problems and also presenting solutions and even presenting alternative models too. But, it is also true that despite of its various weaknesses, still CAPM could not be totally abandoned as useless. Finance still need CAPM, dead or alive.

Key Words: Capital Asset Pricing Model (CAPM), Utility Theory, Prospect Theory, Market Portfolio, Hyperbolic Absolute Risk Aversion (HARA).

1. Introduction
For decades researchers are debating the most popular and widely used asset pricing model-Capital Asset Pricing Model or commonly known as CAPM (Graham and Harvey (2001) and Brounen, Abe de Jong and Koedijk, (2004)), some rejecting it altogether both on theoretical as well as empirical grounds but mostly researchers have given plastic surgery to it. Criticism has often been misunderstood and the borderline between theoretical and empirical problems seems missing especially by beginners in financial econometrics. Not all criticism has its root in CAPM. In addition to major assumption of frictionless market in a mean-variance efficient portfolio framework of Markowitz’s Portfolio Theory (1952), Sharpe (1964) and Lintner (1965) added unlimited lending and borrowing and homogenous beliefs to hold the equilibrium conditions in CAPM. In other words, mostly theoretical criticism of CAPM is hence of Portfolio Theory and vice versa.

In other words, mostly theoretical criticism of CAPM is hence of Modern Portfolio Theory (MPT) and vice versa. Purpose of this paper is not to defend CAPM or MPT, without prejudice, but identify areas of debate, primarily theoretical at times supported by empirical evidence, of academicians and their contributions, and areas for future research. The paper is one of it’s kind as it covers those theoretical problems of CAPM which are not generally discussed in academic
literature and its remedies previously lying in highly adept books of financial economics and dispersed in different research papers making difficult to give these theoretical problems of CAPM a body in the mind of the reader and leaves out those problems that are frequently discussed and readily available\(^\circ\). The paper tries to strike the balance by addressing the theoretical issues but never letting the reins of simplicity for beginners in financial economics and advance portfolio analysis and investment. We also contest that though CAPM might be at times empirically weak but it can not considered as invalid and if modified, its performance can be enhanced.

The paper is divided into four sections. Section (1) gives an introduction to the paper and section (2) presents a brief introduction of MPT and CAPM. Section (3) covers the theoretical problems of CAPM and their suggested solutions. As CAPM heavily depends on Markowitz’z MV-framework so a criticism directed to MV is involuntarily criticizes CAPM. In this section, criticism based on theoretical framework is discussed and covers utility theory, prospect theory, Roy’s safety first principle, rational expectations and normality condition. The empirical issues are not argued upon as they are not the primary domain of this paper, although, as a response to the criticism, empirical support has also been presented. Section (4) is conclusion and recommendations.

2. An Introduction to CAPM-The Menace
CAPM, independently developed by Sharpe (1964), Lintner (1965), Mossin (1966) & Treynor (1962)\(^{16}\), is based on Markowitz’s MV-Approach (1952) under conditions of market equilibrium. Initially, Portfolio Theory is based in maximizing return (E). Markowitz argues that dependence on alone return (E) means that diversifiable portfolio, in no case, is preferred over all non-diversifiable portfolios. So he includes variance (V) as a criterion for portfolio selection and concludes that the attainable set depends on E,V and covariance (correlation) of assets generating the efficient frontier of portfolios in a mean-variance (M-V) framework thus selected-laying the foundation of Modern Portfolio Theory.

Tobin (1958) presents 2-Fund Separation in his Liquidity Preference Theory and suggests a course to identify the appropriate portfolios among the efficient set. 2-Fund Separation Theorem states that each investor will have a utility-maximizing portfolio that is a combination of risk-free asset and a portfolio (or fund) of risky assets, that is determined by the capital market line (CML) tangent to the investor’s efficient set of risky assets and if the assumptions are met then equilibrium conditions are satisfied, we then have the 2-fund separation. Both Markowitz’s and Tobin’s work is theoretical based on Mean-Variance Behavior (MVB) until Sharpe (1964) made it empirically testable.

According to Markowitz (1952), apart from systematic risk all other (or idiosyncratic) risk can be diversified away which is measured by the standard deviation of returns. CAPM employs that for more expected risk investor desire more expected return in a linear relationship between risk and return on efficient portfolios as depicted by Capital Market Line (CML). According to CML,

\(^{15}\) Like unlimited lending and borrowing, short sales allowed and effect of taxes.
\(^{16}\) Sharpe ultimately won noble prize in 1990 for this work but arguably Treynor is the pioneer for CAPM. However the first paper “Toward a Theory of Market Value of Risky Assets” was written by Treynor in CAPM in 1962 was not published until 1999 (French, 2003).
the investor is compensated for delayed consumption as sum of the return for delaying consumption and a premium for bearing the risk inherent in the portfolio in one-period investment decision. CAPM can be graphically represented using Security market line (SML). It displays the expected rate of return of an individual security or portfolio in terms of risk-free rate and relative risk of a security or portfolio (Haugen (2001), Elton, Gruber, Brown and Goetzmann (2003) and Galagedera (2007)).

3. Exclusive Theoretical Problems of CAPM-Great Expectations
CAPM heavily relies on MPT which in turn on Expected Utility Theory (EUT). Much of critique on CAPM eventually falls in domain on EUT. So an effort has been made to address issues concerning EUT and what academicians have recommended upon. We will start with an introduction of EUT and then continue with the criticism associated with it.

EUT, initially proposed by Nicholas Bernoulli (1713) and solved by Daniel Bernoulli (1738) and later adapted by John von Neumann and Oskar Morgenstern in game theory (von Neumann–Morgenstern utility theorem, 1953), deals with consumption and investment decision of an individual in which the investor is rational who determines the possible payoffs and assigns an index to each possible outcome and chooses the one which maximizes the expected value of index. An investor maximizes using a preference relation having a utility representation. Generally there are two approaches for preference relation; objective (von Neumann–Morgenstern utility function, 1953) and subjective (Savage, 1972), and as MPT is based on von Neumann–Morgenstern utility function17, so the issues will be discussed from this perspective following Huang and Litzenberger (1988).

3.1 EUT is Rejected Altogether-Live, Tell Truth and Shame the Devil
von Neumann–Morgenstern’s (1953) EUT is directly contradicted by Kahnemen and Tversky’s (K&T) Prospect Theory (PT) (1979) and its modified version Cumulative Prospect Theory (CPT) (1992)18 which implies that EUT is not valid thus making MPT and CAPM questionable. CPT uses change of wealth and decision (DW) instead of total wealth and objective probabilities used in EUT. If CAPM has to hold then EUT has to hold rejecting CPT altogether. However Levy (2006, 2010) argues that a rejection of EUT does not necessarily means a rejection of CAPM by using First degree Stochastic Dominance (FSD)19, which is derived from EUT framework, constructs DW as introduced by CPT and taking precaution that they do not violate FSD. He proves that all investors choose investments along CML and concludes that though the optimal selected portfolio under MV and CPT are different but Separation Theorem remains intact thus CAPM holds even under CPT methodology.

17 There are four axioms of von Neumann–Morgenstern rationality namely; completeness, transitivity, continuity, and independence. Instead of continuity, an alternative axiom can be assumed that does not involve a precise equality, called the Archimedean property (Neumann and Morgenstern (1944)). These axioms define the rational investor crucial for MPT and CAPM (for details see Huang and Litzenberger (1988)).

2 Kahneman eventually won noble prize in 2002 in field of economics.

19 FSD suggests that investor prefers more than less. SSD (second degree stochastic dominance) add up the investor’s risk aversion and TSD (third degree stochastic dominance), in addition to the first two conditions, state that investor’s risk aversion is at a decreasing rate (Elton, Gruber, Brown and Goetzmann (2003)).
In addition, under the assumption of normally distributed returns, Levy, Giorgi and Hens (2004) have shown that the CPT of Tversky and Kahneman (1992) is consistent with the CAPM in a way that, for financial market equilibrium, the SML Theorem holds. And lastly, while CPT can rationalize the equity premium puzzle when the extreme risk seeking implied by CPT are mitigated but it fails to explain the size and value premium puzzles (Giorgi, Hens and Post (2005)). This suggests that though CPT differs from CAPM but CPT can not invalidate CAPM altogether.

3.2 EUT is Criticized But Not Rejected Altogether—Good Riddance
EUT contains three major behavioral axioms namely; preference relation, substitution axiom and Archimedean axiom, of which substitution axiom is violated in empirical experiments as identified by Allais (1953) in his famous Allais Paradox. In Allais Paradox, individuals depict behavior that is in violation of substitution axiom. Machina (1982) argue that if the conditions that individual’s preference can be represented by a utility functions and the function can be differentiable in certain sense can be then EUT can be carried out despite substitution axiom is being violated. Levy (2010) proposes a different solution, if DW is used and normal distribution assumed, all investor’s selected portfolio are located on CML—implying that CAPM remains intact.

3.3 Safety First (SF) is better than EUT—Defense is best offense
Roy (1952) argues than SF-rule or risk of disaster is the main contributing factor as to portfolio and investment decision-making rather than M-V framework and if accepted then it makes CML and CAPM invalid. Even Markowitz (1959), recognizing the importance of SF-rule or in other words the downside risk, provides two suggestions that downside risk can be measured as below-mean semivariance (SVm) and below-target semivariance (SVt) which he calls them partial or semi-variances. However he stays with simpler variance rather than semivariance as using a cosemivariance matrix require twice the number of data inputs than the variance matrix making it mathematically complex until the advent of computers in 1980s (Markowitz (1959), Nawrocki (1999)).

Mean semivariance or target semivariance has better explanatory compared to M-V framework when the distribution is asymmetric (not normal or is skewed) with fat tails (kurtosis) while in case of normal distribution both variance and semivariance yield the same results (Nawrocki (1999) and Estrada (2002)). In the 60s and 70s, research on semivariance continues with studies by Quirk and Saposnik (1962), Mao (1970), Klemkosky (1973) and Ang and Chua (1979)20 demonstrating superiority of the semivariance (mean or target) versus the variance. But the question arises that stochastic dominance, very powerful risk analysis tool, used to measure semivariance can not be quantified. In 1975 Bawa makes the breakthrough. He generalizes semivariance measure of risk with the development of the Lower Partial Moment (LPM), proxy for stochastic dominance, to reflect less restrictive class of decreasing absolute risk aversion (DARA) utility function—one of three classes of risk measure developed by Pratt (1964)-Arrow (1965)21. Fishburn (1977) extends the general LPM model and provides unlimited view of LPM.

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20 Using performance measures developed, notably by Sharpe, Treynor and Jensen.
21 Arrow-Pratt measure of absolute risk-aversion (ARA) can be classified as DARA (decreasing absolute risk aversion), IARA (increasing absolute risk aversion) and CARA (constant absolute risk aversion). Like for absolute risk aversion, the corresponding terms constant relative risk aversion (CRRA) and decreasing/increasing relative risk aversion (DRRA/IRRA) are used (Arrow (1965), Pratt (1964)).
The Fishburn family of LPM utility functions works for all classes of investors; risk averse, risk seeking and risk neutral. This makes semivariance based on SF-rule superior than Markowitz’s M-V framework. There are four major modifications of CAPM based on SF-rule namely; Hogan and Warren (1974), Bawa and Lindenberg (1977), Harlow and Rao (1989), Estrada (2002) and Estrada and Serra (2005). All claim that incorporating downside risk instead of variance yields better results or in other words CAPM or EUT is inferior to Downside-CAPM (DCAPM) based on SF-principle. In addition, Post and Vilet (2004) proves that downside CAPM and conditional downside CAPM performs better than ordinary CAPM and conditional CAPM.

A scrupulous analysis of the disagreement between CAPM and DCAPM yield that it is not to invalidate CAPM but to show DCAPM’s superiority with the strongest rationale that the distribution is not normal and investors are safety conscious. However, Levy (2010) proves that though CAPM does not hold when distribution is non-normal but modified version of CAPM (Zero-beta CAPM) does hold. Nawrocki (1999) contests that for different distributions CAPM is a special case where results of CAPM and downside risk incorporated in CAPM yields similar results—thus validating CAPM despite the SF-rule and monthly returns do not seem significantly depart from normality in developed markets. Estrada (2001, 2002) concludes that for highly volatile stocks downside risk is not a good measure and other risk measures are to be included. Still more Post and Levy (2008) assert that if investors are risk averse for losses and risk seeking for gains, then they are willing to pay a premium for stocks that give downside protection in bear markets and upside potential in bull markets. As to Fishburn family of LPM utility functions working for all classes of investors; risk averse, risk seeking and risk neutral, Levy (2010) proves that also CAPM holds for risk-seeking investor but with an additional constraint on borrowing, say that it shall not be x-times more than initial wealth, to guarantee equilibrium and all optimal portfolios are on CML.

3.4 Rational Expectations-Pandora’s Box!

The unobservability of ex ante expected returns and betas leads researchers to use instead ex post. To solve this problem it is assumed that investor has rational expectations i.e. an investor’s expectations are fulfilled ex post or in other words rational expectations asserts that outcomes do not differ from expectations. Rational expectations are basis of efficient market hypothesis (EMH) while contradicting it, on the other side, is behavioral finance (BF). CAPM is based on EMH and in turn on rational expectations. A rejection of EMH implies CAPM as invalid. To study the assumption of rational expectations one has to know both the background of EMH and BF.

Rational expectation is originally proposed by John F. Muth (1961) and later their earliest and most striking application is EMH of asset prices. Harry Roberts in his paper “Stock Market ‘Patterns’ and Financial Analysis” (1959) is the foremost presenting the idea of rational investor

22 Harlow and Rao (1989) and Estrada (2002) both have set individual investor’s return as target return with difference as Harlow and Rao (1989) define covariance between two securities to be same (e.g. i and j as same as between securities j and i) while Estrada (2002) take covariance between securities i and j is different from that of between securities j and i.

23 EMH asserts that financial markets are “informationally efficient”. That is, one cannot consistently achieve returns in excess of average market returns on a risk-adjusted basis, given the information publicly available at the time the investment is made. There are three major versions of the hypothesis: "weak", "semi-strong", and "strong" (Haugen, R.A. (2001)).
in asset pricing. Eugene Fama (usually the term “efficient market” is attributed to him) in his 1965 paper, “Random Walks in Stock Market Prices,” concludes that stock market reflects all past information incorporated in current prices implying that the cause is market efficiency based on investor’s rationality. With the advent of “event study” methodology in 60s gives boost to methodological approach to EMH and throughout in 70s EMH ruled and so did CAPM and MPT with only minor resistance.


The introduction to EMH and BF is to comment which one is better but to emphasize the importance of the assumption of rational investor and how much profound is the disagreement between EMH and BF. CAPM, based on rational investor, and when tested on ex post is rejected (Black-Jensen-Scholes (1972), Fama and MacBeth (1973), Fama and French (2004)), however when tested on ex ante CAPM is not rejected (Levy (2008, 2010)). To take ex post and ex ante as same will irony to CAPM. Secondly, the substitute EMH is BF which in turn relies heavily on PT and CPT. PT and CPT have been discussed in detail and its implication to CAPM likewise

24 First event study was designed and conducted by Eugene Fama, Lawrence Fisher, Michael Jensen, and Richard Roll in 1969 in their article “The Adjustment of Stock Prices to New Information”, a study of stock market reaction to announcements of stock splits.


(see Section II-I). And more importantly million dollar question arises as to what is substitute to rational expectations! The answer perhaps lies in adaptive expectations, recently being developed by Lo (2004, 2005) as Adaptive Market Hypothesis (AMH) at MIT. Though AMH is claimed to be a replacement of EMH but it relies on CPT and also have things common with EMH thus opening a new Pandora’s box.

3.5 Normality-A Case of Mistaken Identity!
Quadratic utility or normal distribution in a symmetric environment is assumption of CAPM to determine the efficient frontier based on MV-framework. According to MV-framework, random variables are linearly characterized by the first two moments; mean and variance irrespective whether the distribution is characterized by third or fourth moment or not. Of the two, quadratic utility feels the maximum heat. It is criticized as the quadratic concave utility curve turns down after they reach the “bliss point” which is obviously unrealistic assumption for wealth. The second criticism that stands for both quadratic utility and normal distribution is that it leaves no room for higher moments and relies on the first two moments only. Quadratic utility also assumes IARA and IRRA. Conversely, Blume and Friend (1975) find evidence for CRRA for investors’ behavior while Cohn et al. (1975) show that investors exhibit DARA while quadratic utility cannot exhibit strictly DARA-essential for risk aversion (see Chavas (2004), chapter 4). Another blow came from Kahemmann and Tversky’s PT which contests that investors depict different behavior in different circumstances and can not only be tied to risk averseness as is done in quadratic utility function.

For expediency, researchers prefer normal distribution as compared to quadratic utility function. Nevertheless normal distribution assumption is a sufficient not a necessary condition to derive CAPM assumed for statistical purposes and after all CAPM is a model and in order to validate it has to be tested-empirically. Perhaps the strongest evidence in favor of MV-framework (based on quadratic utility) is Levy and Markowitz (1979) who show empirically that the M-V rule is an excellent approximation to expected utility. The utility loss induced by adopting this approach has been found empirically to be negligible (see also Kroll, Levy and Markowitz (1984) and Markowitz (1991)).

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27 Adaptive expectations means that people form their expectations about what will happen in the future based on what has happened in the past.
28 His two papers in this regard can be regarded as landmark, namely: Reconciling Efficient Markets with Behavioral Finance: The Adaptive Markets Hypothesis and The Adaptive Markets Hypothesis: Market Efficiency from an Evolutionary Perspective.
29 “Bliss point” is a quantity of consumption such that further increases would make the consumer less satisfied (B. Binger and E. Hoffman (1997)).
30 Expanding using Taylor series around mean and taking expectation, the higher-order term other than the second derivative are zero for quadratic utility and in case of normal distribution the odd terms are equal to zero and the even terms are function of variance only (for details see Cuthbertson and Nitzsche (2004)).
32 Both Nielsen (1990) and Allingham (1991) provide sufficient conditions for existence of CAPM equilibria. However, these conditions are difficult to interpret within the expected utility paradigm because both authors assume that all agents have preferences over mean and variance alone. Also see Berk (1997).
33 Included as finite-sample properties of asset pricing model test are difficult to derive (Campbell, Lo and MacKinlay (1997), chapter 5).
However this phenomenon has been attributed to that they are driven by an elliptical distribution\(^{34}\), for which the mean-variance approximation of the expected utility for all utility functions remains exact (Chamberlain, 1983). In contrast, under large departure from normality, in particular when the distribution is severely asymmetric, MV-criterion may fail to correctly approximate the expected utility (see Chunachinda et al. (1997), Athayde and Flôres (2004) and Jondeau and Rockinger (2006)). A large body of literature is available that MV-framework, on which CAPM rests, lacks theoretical support and stresses higher moments (skewness\(^{35}\) and kurtosis) are to be included\(^ {36}\). This criticism has gained momentum in the last two decades and research has been discreetly diverted towards other solutions notable by incorporating Roy’s safety first concept into downside risk (already discussed in Section 3.3)\(^{2}\).

As an alternative to quadratic utility, log-normal utility can be used which significantly performs better\(^ {37}\). For positive skewness, log-normally utility captures stock returns more usefully\(^ {38}\) and exhibits both DARA and CRRA\(^ {39}\). Still, to provide more flexibility, more appealing hyperbolic absolute risk averse (HARA) class can be used. The HARA class is of great importance in finance and macroeconomics. It is a convenient class of utility functions, simple enough and gives sufficient structure to ease theoretical and empirical work, but at the same time it encompasses all major specifications found in the literature\(^ {40}\). HARA class makes MV-framework more flexible and saving CAPM from the onslaught in the name of quadratic utility and rather than rejecting CAPM altogether, making CAPM more viable to modifications without altering the basic theoretical foundation of the most popular asset pricing model in the world (see for example Hamada and Valdez (2008)).

4. Conclusion
CAPM, most popular and used asset pricing model in the world, is not free of controversies. Both these controversies are, many a time, mixed up and the borderline between theoretical and empirical problems is not properly identified. Though some believe that the solution lie in the empirical arena, on the other hand some believe that CAPM is theoretically invalid. But the reality is that both hold the key and neither one can be overlooked. Most of literature regarding CAPM is based empirically while the theoretical problems of CAPM discussed are random, dispersed in literature and complicated for starters in the field of financial economics and advance portfolio management.

\(^{34}\) Elliptical distribution has many classes and the famous among them are normal, student t, logistic, laplace and uniform distribution. All elliptical distributions are symmetrical and their use in portfolio is essential and widespread. For details consult Chamberlain (1983) and Owen and Rabinovitch (1983).

\(^{35}\) For details Teall (1999), chapter 7.

\(^{36}\) Kraus and Litzenberger (1976) and Harvey and Siddique (2000) assume a cubic utility function (or a third-order Taylor series approximation to the true utility function) to incorporate skewness.

\(^{37}\) For details see Haugen, R.A. (2001).

\(^{38}\) Log, power, exponential, and quadratic all belong to HARA class (for a detail account of it see Lengwiler (2004), chapter 4).

\(^{39}\) As HARA covers DARA, CARA, IARA, DRRA, IRRA and CRRA (for a detail account of it see Lengwiler (2004), chapter 4).
Most of literature regarding CAPM is based empirically while the theoretical problems of CAPM discussed are random, dispersed in literature and complicated for starters in the field of financial economics. So problems having origin in EUT, PT and CPT are discussed in addition to Roy’s safety-principle. Though an empirical problem, the condition of normality is also discussed as more often it is taken as theoretical overlooking it being a sufficient condition. In all cases CAPM holds, but at times it can be argued that if modifications can be made other versions of CAPM are more empirically valid. The disagreement between EMH and BF and MV-framework and downside risk let the person hold breath and think which to reject or prefer. Perhaps the answer lies in modified versions.

Due seductive simplicity of CAPM and its ability to teach the relationship between risk-return and pricing of financial assets, Academicians had and will continue to teach the CAPM in business schools but all has not to change but the change itself. Rather than utterly invalidating CAPM, complex model like Intertemporal CAPM presented by Merton (1973) working both on MV and downside risk framework, Consumption-based CAPM and Consumption-based Downside-CAPM and Continuous-time equilibrium CAPM-based models are the key to asset pricing. These models have assumption similar to CAPM and thus can gain world acceptability. At the same time it is also important to warn people concerned about the empirical problems which probably invalidate the use if these models and at times CAPM itself in applications.
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