Dynamic Stochastic General Equilibrium Models: A Tool for Monetary Policy in Light of Lucas Critique

Syed Kashif Saeed  
COMSATS Institute of Information Technology, Islamabad, Pakistan

Shahid Mehmood Sargana  
COMSATS Institute of Information Technology, Islamabad, Pakistan

Usman Ayub  
COMSATS Institute of Information Technology, Islamabad, Pakistan

Faisal Nawaz  
COMSATS Institute of Information Technology, Islamabad, Pakistan

Abstract

Dynamic stochastic general equilibrium (DSGE) models are the macro models based on Micro foundations. These are playing an important role in conducting monetary policies at various central banks like Norges Bank, bank of England etc. The primary emphasize of these models is on expected future outcomes while determining current outcome. Due to inclusion of Micro foundations and expectations these are in line with Lucas Critique. Although very popular in academia and policy makers still these models are in transitional phase to prove their usefulness in forecasting and policy making.

Keywords: Dynamic stochastic general equilibrium, Micro foundations, Euler Equation, output gap.

1. Introduction

The last two decades can be called era of scientific revolution which has resulted in the gigantic development of mathematical and statistical tools resulted in the enormous change in the discipline of macro economics. The said development has changed the approach of economists to look at problems and also provide the tools for solving theoretical issues which were, previously, out of questions due to lack of technology required.

Besides the tremendous improvement in research tools, past two decades have also full of economic crises, starting of Tequila Crisis in 1995, Asian Financial crisis in 1997 and latest Global Financial Crisis in 2008 etc. Two important questions have been raised after these crises. First, are our economic policies (i.e. Monetary and fiscal) sufficient to handle these crises and second are our economic models capable in forecasting and mitigating the adverse effect of these crises.

Economic literature places monetary policy at important level while discussing both of these questions. A lot of research has been conducted for designing optimal monetary policy [Taylor, 1993; Woodford, 2003; and Clarida et al 1999, 2000; Monacelli, 2005].
Lucas (1976) argues that while preparing the macroeconomic policy and predicting its effect entirely on the grounds of historic data can lead to disastrous or at least ineffective macroeconomic policies. This argument is known as Lucas critique in economics literature. His own words are as follows:

“Given that the structure of an econometric model consists of optimal decision rules vary systematically with changes in the structure of series relevant to the decision maker, it follows that any change in policy will systematically alter the structure of econometric models” (Lucas, 1976, p.41)

Lucas explains his view and poses a question also on the effectiveness of very large country wide macroeconomic models. He argues that if we are interested in estimating the impact of policies then our parameter should be “Deep parameter”, i.e. these should be “policy invariant”. He calls these as Structural parameters which should be based on such variable which relates to individual preference, resource constraints, technology improvement etc. After this, policy makers will be able to predict that how individual will behave, firms will react and aggregation of individual and firm decision will lead us to estimate the macroeconomic impact of any suggested economic policy.

It is the great contribution of Lucas to motivate macroeconomists towards micro foundations for their models. Kydland and Prescott (1977) also stressed on importance of micro foundations especially individual (Rational) expectations. They argue in their own words as:

Even if there is an agreed-upon, fixed social objective function and policymakers know the timing and magnitude of the effects of their actions, discretionary policy, namely, the selection of that decision which is best, given the current situation and a correct evaluation of the end- of-period position, does not result in the social objective function being maximized. The reason for this apparent paradox is that economic planning is not a game against nature but, rather, a game against rational economic agents. …… there is no way control theory can be made applicable to economic planning when expectations are rational. (Kydland and Prescott ,1977)

The need for optimal monetary policy design in the light of Lucas critique, availability of mathematical and statistical tools with gigantic computing power, combinely, resulted in evolution of Dynamic Stochastic general Equilibrium (DSGE) models.

This short paper intends to provide very brief orientation towards Micro foundation based Macroeconomic models. The paper run as follows. Section 2 discuss about basic concepts of Dynamic Stochastic General Equilibrium (DSGE) model. Section 3 provides micro foundation for DSGE model whereas section 4 discusses various available methodologies for simulating, calibrating and estimating DSGE model and section 5 concludes.

2. What is DSGE Model?
DSGE is the abbreviation of Dynamic Stochastic General Equilibrium model containing three important jargons requiring need to explain. First is the Dynamic, second is Stochastic and last is General Equilibrium.
Dynamic analysis relates to decision made at the respective time period. Decisions made at time t for t+1 can be on the basis of Rational Expectation or Adaptive Expectations, as shown in Figure 1.

**Figure 1: Dynamic Analysis**

Therefore Dynamic aspects of the model stresses that today’s outcome are determined on the basis of future expectations. DSGE models highlight the importance of expectations in managing economy. It can be said that to manage trade-off between inflation and output what Central bank need to do is to manage expectations.

Stochastic Analysis relates to random shocks which affect any economy and then their impact is propagated throughout some future time period. With the help of these random shocks uncertainty are injected into economic system otherwise economy would be running as predicted without any uncertainty or fluctuations. These random shocks can be supply shocks, demand shock etc.

**Figure 2: Stochastic Analysis**

General equilibrium analysis relates to understanding model dynamics after taking into the consideration of three relevant sectors. These three relevant blocks/sectors for DSGE models are Supply side of economy, Demand side of economy and the Monetary Policy. Models regarding each block are based on explicit assumptions of major economic agents in each block i.e. household, firms and monetary authorities.
Thus DSGE model are dynamic model incorporating stochastic components involving all relevant sectors of economy.

3. Micro Foundation of DSGE Model

As explained already that in last twenty five years, macroeconomic modeling has made enormous development in all aspects (both empirics and theory). With the help of Micro foundation, DSGE models have made it be possible to escape or at least minimize the Lucas (1976) critique.

The distinctive feature of DSGE models is that decision rules of economic agents are derived, from assumptions about preferences and technologies, by solving intertemporal optimization problems. The uncertainty present in model is generated by exogenous stochastic processes or random shocks that disturb demand side and/or supply side of the economy and resulting deviations from interest-rate feedback rule of central bank.

DSGE model, generally, are composed of three sector that is Household, Firms and monetary authorities. The discussion of how microeconomics of each sector contributes towards baseline model is given below. Basic Notation of model has been adapted from Walsh (2003) and Gali and Moncelli (2005).

a) Households’ Decision and Demand Side of Economy

An inverse relationship between output and real interest rate is an integral component of demand side of economy. This inverse relationship is result of pattern in household consumption decisions. A rational representative household, of an economy always intends to maximize his utility by trading off his consumption ($C_i$) and leisure ($N_i$). The preferences for such a household can be described by an inter-temporal Constant Relative Risk Aversion utility function (CRRA) as

\[
U = E_t \sum_{i=0}^{\infty} \beta_i U(C_i, N_i) \\

U = E_t \sum_{i=0}^{\infty} \beta_i \left[ \frac{C_{t+i}^{1-\sigma}}{1-\sigma} - \frac{N_{t+i}^{1+\omega}}{1+\omega} \right]
\]  

(1)

Where, $\beta_i$ Inter-temporal discount factor, describes the rate of time preferences, $\sigma$ represents the inverse of elasticity of Inter-temporal substitution in consumption, also gives the degree of relative risk aversion and $\omega$ is the inverse of wage elasticity of labor supply. $\sigma > 0$, $\omega > 0$ and $\beta_i \in (0,1)$. $C_i$, the composite consumption index of foreign and domestically produces goods is defined as
\[ C_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} \left( C_{H,t} \right)^{\frac{\eta-1}{\eta}} - \alpha^{\frac{1}{\eta}} \left( C_{F,t} \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \]  

(2)

Where \( \eta \) is the price elasticity of substitution between home and foreign goods is, \( \alpha \in (0,1) \) is the trade share, also measures the degree of openness. \( C_{H,t} \) is an index of consumption of domestic goods and \( C_{F,t} \) is an index of consumption of imported (foreign) goods.

Rational households under restriction of limited budget try to minimize the cost of consumption.

\[ PC_t + E_t \left( Q_{t,j,t+1} D_{t+1} \right) \leq D_t + W_t N_t + T \]  

(3)

\( W_t N_t \) is the Nominal wage times the number of hours worked, \( D_t \) is the payoff from securities held and \( T \) are the net of taxes and transfer payments. So Left hand side is providing us the expenditure whereas Right Hand side is sources of Income.

Applying Lagrangian optimization function to maximize eq (1) subject to eq (3) will result in

\[ C_t^{-\sigma} = \beta (1 + i_t) P_t E_t \left[ C_{t+1}^{-\sigma} \right] \]

Above equation is also called Euler Consumption Equation for the optimal inter-temporal allocation. So as per Euler equation, when real interest rate increases or future consumption is expected to decrease or discount rate increases, current consumption is expected to decrease.

Log-linearizing equation above equation and applying the concepts that marginal rate of substitution between leisure and consumption is equal to real wage, results in

\[ \hat{c} = E_t \hat{i}_{t+1} - \frac{1}{\sigma} \left( \hat{i}_t - E_t \pi_{t+1} \right) \]

In above equation, \( \hat{i} \) is the short term nominal rate and \( \pi_t \) is the consumer price inflation rate.

Assuming the closed economy model, one can safely assumes that \( y = c + g \), and also denoting output gap as \( x = y - \hat{y} \), then above equation can be shown as follows;

\[ x_t = -\phi [i_t - E_t \pi_{t+1}] + E_t x_{t+1} + e_t \]  

(4)

Where \( \phi = \frac{1}{\sigma} \), \( g_t \) is the disturbance term which is AR(1) process \( e_t = \mu e_{t-1} + \eta_t \).

Equation (4) is similar to traditional IS equation and therefore it is also called Demand side equation in DSGE model.

**b) Firms’ Decision**

Supply side of the economy in DSGE model incorporates the behavior of the firms in setting up their prices keeping in view their demand as well. Rational entrepreneurs at various firms, in the same way, intend to maximize the profits of firms. Conventionally the profit maximization firms used to consider the restriction from demand and production side. The concept of price rigidities provided another dynamics to firm’s optimization decision. Now it is assumed that a certain percentage of business don’t change the prices whereas remaining percentage does change prices\(^1\). To incorporate the price rigidity in production function, Calvo prices (1983) are introduced.

For simplicity, it is assumed that labor is the single factor of production. Thus \( Y_{j,t} = Z_t N_{j,t} \)

---

\(^1\) This phenomenon has been discussed at length, please see Bils and Klenow (2004); Nakamura and Steinsson (2008)
$Z_t$ is the country specific aggregate productivity disturbance which is assumed to be stochastic with constant returns to scale, that is, $E(Z_t) = 1$. This can also be called as Productivity shock.

Aggregate output can be defined as

$$Y_t = \left[ \int_0^1 Y_j(j) \cdot (1 - (1 - \rho) dj \right]^{1/(1 - \rho)}$$

$$\ln Y_t = \left[ \int_0^1 \ln Y_j(j) \cdot (1 - (1 - \rho) dj \right]^{1/(1 - \rho)} = \ln(Z_t) + \ln(N_{j,t})$$

$y_t = z_t + n_t$

Firm $j$ minimize cost subject to producing the firm specific good $Y_{j,t}$

$$L = \left( \frac{W_{j,t}}{P_{D,t}} \right) N_t + \phi_t(Y_{j,t} - Z_t N_{j,t})$$

First order condition gives

$$MC_t = \frac{\partial L}{\partial p_t} = \frac{W_t}{P_{D,t}} Z_t$$

$$mc_t = \tilde{W}_t - \tilde{P}_{D,t} - \tilde{z}_t$$

So we can see the relationship among real marginal cost, labor factor productivity and real wages.

Assuming Calvo price assumption, only $(1 - \theta)$ firms revising prices thus, $\theta$ measures price rigidity. Domestic price index is defined as

$$P_{H,t} = P_{H,t} \left( j \right) \left( \frac{P_{H,t-1}}{P_{H,t-2}} \right)^{\theta_H}$$

Firms maximize their profits after setting the new price $p^*_{j,t}$ at time $t$ as:

$$\max \sum_{i=0}^{\infty} E_t \left\{ (\theta_i) \Delta_{i,t+i} \left( Y_{j,t+i} \left[ \frac{p^*_{j,t}}{P_{t+i}} - \phi_{t+i} \right] \right) \right\}$$

s.t.

$$Y_{j,t} = \left( \frac{p_{j,t}}{P_t} \right)^{-\epsilon} C_t$$

(5)

(6)

It is intuitive for firms to consider elasticity of demand while prices are being set. Substituting the demand function [equation 6] in maximization function [equation 5] results in

$$\max \sum_{i=0}^{\infty} E_t \left\{ (\theta_i)^j \Delta_{i,t+i} \left[ C_{i,t+i} \left[ \frac{p^*_{j,t}}{P_{t+i}} \frac{p^*_{j,t}}{P_{t+i}} \right]^{-\epsilon} - \phi_{t+i} \left( \frac{p^*_{j,t}}{P_{t+i}} \right)^{-\epsilon} \right] \right\}$$

Taking First order condition and applying algebra will result in
\[ P_{H,t}^* = \left( \frac{\varepsilon}{1 - \varepsilon} \right) \frac{E \sum_{i=0}^{\infty} (\beta \theta_i)^i (C_{t+i})^{1-\sigma} \varphi_{t+i} (P_{t+i})^\varepsilon}{E \sum_{i=0}^{\infty} (\beta \theta_i)^i (C_{t+i})^{1-\sigma} (P_{t+i})^{\varepsilon-1}} \]

\[ P_{j,t}^* = \left( \frac{\varepsilon}{1 - \varepsilon} \right) \frac{E \sum_{i=0}^{\infty} (\beta \theta_i)^i (C_{t+i})^{1-\sigma} \varphi_{t+i} \left( \frac{P_{t+i}}{P_t} \right)^\varepsilon}{E \sum_{i=0}^{\infty} (\beta \theta_i)^i (C_{t+i})^{1-\sigma} \left( \frac{P_{t+i}}{P_t} \right)^{\varepsilon-1}} \]  

(7)

The above equation describe rule for optimal price; and \( \left( \frac{\varepsilon}{1 - \varepsilon} \right) \) is the markup. Price index consisting of new price and Calvo pricing can be shown as

\[ P_t^{1-\varepsilon} = (1 - \theta_t) P_{j,t}^* + \theta_t P_t^{1-\varepsilon} \]

In above equation, we have written price index as weighted index of newly set price \( P_t^* \) and price index from previous period \( P_{t-1} \).

Where \( P_t = \left[ \int_0^1 P_t^{1-\varepsilon} \, dj \right]^{1/(1-\varepsilon)} \)

Taking log of the above equation and transforming into steady state results in

\[ p_t = (1 - \theta_t) p_{j,t}^* + \theta_t p_{t-1} \]  

(8)

The above equation shows the general price level in steady state which is weighted average of the firms which adjust their prices each period, \( (1 - \theta_t) \) and \( \theta_t \) firms do not adjust their price.

\[ 1 = (1 - \theta_t) \left( \frac{p_{j,t}^*}{P_t} \right)^{1-\varepsilon} + \theta_t \left( \frac{P_{t-1}}{P_t} \right)^{1-\varepsilon} \]  

(9)

Now Equation (6) about firms’ optimal price setting rule can be written as

\[ E_i \sum_{k=0}^{\infty} (\beta \theta_i)^k (C_{t+k})^{1-\sigma} \left( \frac{P_{t+k}}{P_t} \right)^{y-1} \]  

\[ f_t = \mu \left[ E_i \sum_{k=0}^{\infty} (\beta \theta_i)^k (C_{t+k})^{1-\sigma} \left( \frac{P_{t+k}}{P_t} \right)^{y} \right] \]  

(10)

Applying Taylor series approximation and restricting it to two periods the above equation can be written as

\[ \hat{\pi}_t = (1 - \theta) \hat{\varphi}_t + \theta \beta (E_t \hat{\varphi}_{t+1} + E_t \pi_{t+1}) \]  

(11)

\[ \left( \frac{\theta}{1 - \theta_t} \right) \pi_t = (1 - \theta) \hat{\varphi}_t + \theta \beta [\left( \frac{\theta}{1 - \theta_t} \right) E_t \pi_{t+1} + E_t \pi_{t+1}] \]  

(12)

\[ \left( \frac{\theta}{1 - \theta_t} \right) \pi_t = (1 - \theta) \hat{\varphi}_t + \theta \beta [\left( \frac{1}{1 - \theta_t} \right) E_t \pi_{t+1}] \]  

(13)

\[ \pi_t = \lambda \hat{\varphi}_t + \beta E_t \pi_{t+1} \]  

(14)

Where \( \hat{\varphi}_t \) is real marginal cost and \( E_t \pi_{t+1} \) is the expected inflation.

Above equation is the forward looking Philip curve which also describes the supply side of the economy. This forward looking Philip curve is highlighting the relationship between inflation,
economic activity and expectation about inflation. So if economic activity will be higher, higher wages will be paid to labor resulting increase in marginal cost which will ultimately result in rise in inflation.

Now it can be easily understood that how macro model like DSGE are based on micro foundations provided by household behavior and firm behavior. These models are based on coefficients provided through microeconomics variables like Coefficient of relative risk aversion, subjective discount factor, Frisch elasticity of labor supply, fraction of firms that cannot adjust prices, elasticity of substitution between different goods etc. After incorporating these micro foundations, now macro model is expected to be less prone to Lucas critique.

c) Monetary Authorities

After deriving demand side and supply side equation which represents households sector and firm/production sector, now it is important to include monetary authorities into model so that equilibrium can be attained in output-inflation space.

Generally, it is assumed that central banks should follow some rule for setting interest rate thus setting preference on rule rather than discretion. The main objective of monetary policy is to stabilize both output and inflation, so they need to find a tradeoff between two. With this objective they are supposed to follow some rule like Taylor Rule (1993).

The target interest rate for each period is a function of the gaps between expected inflation and output with their respective target levels. Therefore Clarida et al (2000) postulate the linear equation.

$$ R^* \text{ is desired nominal rate when both inflation and output gap are at their target level.} $$

$$ r_t = R^* + \beta \{E[\pi_{t+1} | \Omega_t ] - \pi^* \} + \gamma E[X_{t+1} | \Omega_t ] $$

For incorporating Interest Rate Smoothing, the target rate function can be shown as:

$$ r_t = \rho L(r_{t-1}) + (1-\rho)r_t $$

Where $\beta$ and $\gamma$ are the coefficients for expected inflation rate and output gap; and $\rho$ is the interest rate smoothing factor. After certain algebraic manipulation, the Monetary Reaction function can be shown as

$$ r_t = \rho (L) r_{t-1} + (1-\rho)[\alpha + (\beta - 1)[E[\pi_{t, k} | \Omega_t ] - \pi^* ] + \gamma(x_{t, q})] + \epsilon_t \quad (15) $$

Now our simple and small monetary policy model is complete consisting of Demand side equation, supply side equation and monetary reaction function and whole model is able to, at least partially, answer Lucas Critique. The model’s equilibrium conditions can be summarized by the following equations:

$$ x_t = -\phi [i_t - E_t \pi_{t+1}] + E_t x_{t+1} + e_t $$

$$ \pi_t = \lambda_\phi \pi_t + \beta E_t \pi_{t+1} $$

$$ r_t = \rho (L) r_{t-1} + (1-\rho)[\alpha + (\beta - 1)[E[\pi_{t, k} | \Omega_t ] - \pi^* ] + \gamma(x_{t, q})] + \epsilon_t $$

Following is the canonical representation of the closed economy model. As we can see that net exports and exchange rate are not there in the model. Generally there are two variations to above model closed economy model. First is the open economy model with the assumption when purchasing power parity holds and second is the variation of the above model when purchasing power parity does not hold.

4. Using DSGE Model for Policy Making

After deriving DSGE model, next problematic step is how to use these models for policy making. The issues involve are more computing power and unconventional methods.

Evaluation of DSGE models can be done in primarily three ways, First Calibration, second Simulation and third Estimation. In the initial period of DSGE models, calibration was the most useful way but now its use has been greatly declined mainly due to availability of computational power and
sophisticated econometric methods. Simulation and estimation are greatly in used for all size of models, from small and medium size models used by mainly academia and for large size model mainly used by central banks.

Canova (2007) has extensively discussed different methodologies available for DSGE model. Following methods are being greatly used for estimation of DSGE models;

- Generalized method of moments (GMM)
- Simulated method of moments (SMM)
- Structural VAR based on Minimum Distance
- Maximum likelihood method (ML)
- Bayesian Methods

It is important to note that GMM methods, working as single equation, only use part of the information implied by the model, therefore mostly models use estimation methods other than GMM. Maximum likelihood (ML) method, although generally used, but questioned a lot due to some theoretical issues. For example stochastic singularity problem, the assumption that the model correctly represents the process generating the data up to a number of parameters, its over-sensitivity to model misspecification, identification issues etc. Canova (2007) and An and Schorfheide (2007) has provided enormous discussion on the issue.

In comparison with ML methods, Bayesian methods are supposed to handle these estimation issues for DSGE in better way because in these methods likelihood are augmented with priors about parameter distributions. Due to this augmentation, the posterior distributions provide robust inferences about the parameters. We still cannot say that Bayesian methods are free from flaw but still it is able to answer some questions. Tovar (2008) argues one important point by saying that “the methods for solving and estimating DSGE models are not the standard ones found in the older literature. A good example of this is the econometric methods employed. For instance, Bayesian techniques are not yet a standard part of econometric courses in PhD programmes in economics.”

5. Conclusion
Lucas Critique rightly questions the very basics of every macroeconomic model and it encourages macro economist to think about most important questions that models should not be Policy variant and time inconsistent. Kydland and Prescott (1982) and others took some steps and now we are witnessing DSGE models based on Micro foundations.

DSGE models are undoubtedly based on powerful methods and tools based on various sectors of economy, provide coherent overview and are able to answer changes in structural parameters. But these are not beyond criticism (Sims, 2006). Still its micro foundations are questioned and very importantly its communication power is questioned. But it need to be in mind that DSGE models are in their initial stage and these problem will be discussed in due time. Although academia seems to like DSGE model as its research publication are on increase but only time will tell whether problems related to DSGE outperform its benefits or not.

References